Augmentation Matrix: Manipulating Perceptual Consonance Through Stretched Harmonics

JERICA OBLAK, Ph. D
Composer/Music Theorist
35 West 4th Street,
New York, NY 10003
USA
jerica.oblak@nyu.edu

Abstract: - *Augmentation Matrix* is a compositional system and a modular instrument designed to alter the perception of consonant/dissonant relationships through the proportional stretching of the harmonic series. Since the augmentations are structurally derived from the overtone series, each partial remains a transposition of the original series (found in the given series through the linear functions), while the new primary intervals (found in the given series through the exponential functions) adopt the characteristics of "pseudo-octaves." Utilizing augmentations to build sounds via additive synthesis while structuring compositions with corresponding microtonal mapping/tuning leads to new consonant hierarchies. With practical applications in mind, the system focuses on the augmentations based on tempered primary intervals. It also introduces an organization of "pivot" tones and proposes a method of modulating to different augmented series/sub-series, various transpositions, and the combinations of the two. Although all the augmented series are derived from a single structure, each of them displays a unique harmonic identity and structural characteristics. The resulting augmentations, their modes, chords, harmonic relationships and transpositions can be interpreted independently or in relation to the Western tonal system (as it might be influenced by the harmonic series). *Augmentation Matrix* can be used for sound synthesis and microtonal mapping as well as building melodies, harmony, rhythm, tempo, and/or form. The augmentations also proportionally diminish the microtonal deviations when applied to equal temperament.

Keywords: - Harmonic series, microtonal composition system, spectral music, stretched harmonic theory, consonance/dissonance perception.

1 Introduction

Augmentation Matrix is a modular instrument and a compositional technique developed on the assumption that our consonant/dissonant perception is to a great degree defined by implicitly perceived partials - a guiding web of spectral structures that serves as a reference and a context to our cognition of musical syntaxes. While the corresponding relationships between sound spectra, tuning and musical structuring is statistically difficult to dismiss as a coincidence, it is not to say that musical traditions, including Western tonal music, simply follow the lines of spectral design. It is the narrative of musical events, including their vertical/horizontal sonorities, that provides us with sonic information ultimately triggering our subjective responses. The context of a musical work, cultural conditioning, and psychoacoustic parameters all contribute to our musical perception/conception, and consequently shape the development of musical vocabulary. In *Augmentation Matrix*, I acknowledge the relationship between harmonic spectra and our musical cognition; specifically, I generate a matrix that systematically ties the augmented harmonic series to compositional decisions. The architecture of *Augmentation Matrix* manipulates the sound design as well as the microtonal mapping/tuning by utilizing a simple hypothesis: since the structure of the harmonic series contributes to our perception of consonant/dissonant relationships, it is reasonable to assume that this structure, only augmented (or diminished), would tilt the consonant effect in favor of a new hierarchical order. In other words, "pseudo-octaves" would replace the exponential order of the octaves and stretched (or compressed) series would replace the linear functions of the overtones in a harmonic series. Each partial, in this case an inharmonic one, becomes the beginning of the new transposed series, in this case an augmented one, while the first partial (also enhanced by the amplitude) and all the exponential pairs become new primary intervals, new functional octaves of the augmentations. While all the calculations are microtonally accurate in relation to natural tuning, I prefer to work with the factors of augmentations that result in tempered primary intervals/"pseudo-octaves." Similarly, to William Setharese's *Xentonality* (1998), I match the new sound spectrum with tuning using an electronic medium applying a given augmentation structure to additive sound synthesis and a corresponding microtonal mapping. While acoustic applications of *Augmentation Matrix* disregard precise tunings and the manipulation of the sound spectrum (thus minimizing the perceptual consonance of the augmentation process), they also minimize the microtonal deviations in relation to the tempered system since the augmentations provide theoretically a "higher resolution" in relation to equal temperament.

The aesthetic framework of the resulting series, their modes and harmonic relationships can be defined independently or in relation to the Western tonal system viewed through its presumed relationship with the harmonic series. Although I acknowledge the possibility of linking the two, and I indirectly parallel augmented series with Western tonality, my primary interest in the harmonic series lies in its structure rather than its possible relationship with tonal music. Likewise, my intention is not to apply models of existing harmonic or inharmonic spectra to the structural fabric of my music but rather explore the complex sonorities resulting from the mathematical/theoretical manipulation of the series. I am interested in finding unique characteristics of the individual augmentations within the overall structure common to all the series in the system. The development of the *Augmentation Matrix* has been influenced by spectral music, various microtonal systems created in the 20th century as well as Sethares's research linking tuning and sound spectra with our perception of melodic/harmonic intervals (2005). On the other hand, the actual process of proportional alteration has not been inspired by musical models but rather by visual representational art where employment of such a technique is a common practice. I am particularly interested in augmentations distorting the initial object and I interpret the augmentation process of my system as a distortion of the original overtone series. This paper focuses exclusively on the mathematical aspect of the system and does not deal with the compositional, contextual, aesthetic, perception or performance issues related to the system.

2 Harmonic Series

Since the harmonic series consists of frequencies ascending through the integral multiples of the fundamental,[1] there is a clear relationship between the fundamental and its upper partials. If f_1 indicates the frequency of the fundamental, then the frequencies of its overtones equal $2f_1$, $3f_1$, $4f_1$, $5f_1$, etc. If any of these frequencies are substituted by n, therefore $2f_1 = n$, $3f_1 = n$, $4f_1 = n$, or $5f_1 = n$, etc., it follows that each order of n, 2n, 3n, 4n, 5n, etc. creates a transposition of a given series within the series itself. It means that each harmonic of the harmonic series is a fundamental of a new harmonic series found in the given series through the multiples of n (where n indicates the frequency value of the harmonic or the position of the harmonic in the given series). It also follows that the order of the octaves in the harmonic series is determined by powers of two and if n again indicates the frequency value of the harmonic or its position in the series, then its octave repetitions equal $2n$, 2^2n, 2^3n, 2^4n, 2^5n, etc.

Except for the octave repetitions of the fundamental, the frequencies in the harmonic series are not the pitches of the tempered system used in Western tonal music. After calculating the distance between the first thirty-two harmonics (f_2) and their closest lower fundamentals (f_1), using the formula $c = \log f_2/f_1 \times 3986$, I determined the microtonal

[1] Because each mode of vibration results from a division into some integral number of segments of equal length, the modes of vibration produce frequencies that are integral multiples of the fundamental frequency.

deviations of individual harmonics in relation to equal temperament. That allowed me to express in cents the distances between the successive overtones. Starting with the lowest interval of the harmonic series, I calculated the following order: 1200c, 702c, 498c, 386c, 316c, 267c, 231c, 204c, 182c, 165c, 151c, 138c, 129c, 119c, 112c, 105c, 99c, 93c, 89c, 85c, 80c, 77c, 74c, 71c, 67c, 66c, 63c, 60c, 59c, 57c, and 55c. This order serves as the foundation of my system.

3 Augmented Harmonic Series

Proportional augmentations of the intervals in the harmonic series create an infinite number of augmented series forming my musical system *Augmentation Matrix*. The purpose of the augmentations is twofold: it proportionally diminishes the microtonal deviations when applied to equal temperament and, most importantly, creates various series and sub-series each founded on a primary interval other than the octave thus offering new consonant hierarchies. The relationships between different augmentations and their transpositions are clearly defined by the structure of the harmonic series and the process of the augmentation. The system offers various modulation techniques and analytical models, thus creating the foundation for a new musical syntax.

3.1 Microtonal Deviations

The first feature of the augmentation process relates to the increased microtonal accuracy. Namely, if the half–step units of the tempered scale remain unchanged (in other words, if we use traditional tempered instruments and traditional techniques of playing), it follows that the larger the augmentation of the harmonic series, the smaller the microtonal deviations are in relation to the proportional scale of the harmonic series. The exceptions are smaller microtonal deviations (specifically, microtonal deviations $\leq$ 50c/a, where a is the factor of the individual augmentation) which, when augmented, remain proportionally the same. Since the augmented series are harmonically different from the overtone series, the reduction of microtonal mistakes is more a positive side effect than a practical tool of constructing a microtonally more accurate harmonic series within equal temperament.

3.2 Structure

The second main feature of the system relates to the already mentioned structure of the harmonic series and the relationships among the notes of the series. Since proportions remain the same, it is equally true for the augmented harmonic series as it is for the natural harmonic series, that each note of the series is also a fundamental of a new series, in this case an augmented one. If n indicates the position of the note in a series, it follows that a new series, the transposition of the given augmented series, equals n, 2n, 3n, 4n, 5n, etc. Likewise, the relationship by powers of two (n, 2n, 2^2n, 2^3n, 2^4n, 2^5n, etc.) in the augmented harmonic series always produces equal intervals. Unlike in the harmonic series, these intervals are not perfect octaves, but rather other intervals determined by the initial augmentation (see Figure 2). In each augmented series, the interval determining the above-mentioned order by powers of two, presents the most significant building block of the series and is, therefore, referred to as the primary interval or a “pseudo-octave” of the series. I also believe that it is the most significant element determining our perception of different augmented series. With easier practical application and composition/performance demands in mind, I like to work with augmentations based on tempered primary intervals. For example, when the intervals of the harmonic series expressed in cents are multiplied by 13/12, the primary interval consists of exactly 13 half-steps (m9), see Figure 1; when multiplied by 7/6, the primary interval consists of exactly 14 half-steps (M9); when multiplied by 5/4, the primary interval consists of exactly 15 half-steps (m10); when multiplied by 2, it is exactly two octaves, etc. If primary interval recurrences in the augmented harmonic series can be compared to the octave repetitions in the harmonic series, one can view all intervals as potentially equal. As such, one can replace doublings in octaves with “doublings” in the primary intervals (see Figure 3). So far, I have worked with fifteen different augmentations in my orchestral, chamber and electronic music. Figures 1, 2 and 3 illustrate one such augmentation. As evident from above, the presented augmentation is based on the primary interval of 13 half-steps (m9) and is, therefore, the smallest augmentation in the line of augmented series based on the tempered primary intervals. Therefore, it does not significantly diminish microtonal deviations when applied to equal temperament. Due to its applicable registers, however, it allows one to apply a rather large portion of the series. (See Figure 3, a music example showing the use of the augmented series in my orchestral/vocal piece *Ashen Time*. In this example, I utilize the first 32 notes of the series: fundamentals = C and C1).

3.3 Modulations

There is, of course, an infinite number of possible augmentations. If ignoring extreme registers and counting only the augmented series with tempered primary intervals, one can count 144 augmentations before all the notes of the series are the octave transpositions of the fundamental (the intervals are augmented twelve times). There are 156 augmentations before the order of a given series repeats (the intervals are augmented thirteen times). Since one can choose to modulate from one series to another, it is important to mention exponential relationships between various augmentations. When a given series is multiplied by a positive integer, the new augmented series consists of notes ordered in the given series by the exponent of the same integer. It means that the integral augmentations result in the series consisting exclusively of the notes found in the initial series and might be, as such, viewed as sub-series of the initial series rather then new augmentations. (Of course, when working with augmentations of which primary intervals consist of any number of octaves, the new series will be a sub-series of the harmonic series itself; see example below.) If the factor of augmentation is two and therefore the intervals double in size, the notes of the new series (with the same fundamental) relate to the notes of the initial series by the exponent of two. For example, if we use C2 as a common fundamental and compare the augmented series presented in Figure 1 with the augmented series based on the primary interval of 26 half-steps (M16): C2, D, f+21c, e1, c2+36c, g2+21c, c#3, f#3, a#3+42c, d4+36c, f#4-6c, a4+21c, c5+20c, d#5, f#5-43c, g#5, etc.; in other words, if we compare the series in a 2:1 ratio, we see that the second note of the second series equals the fourth (2^2) note of the first series, the third note equals the ninth (3^2), fourth the sixteenth (4^2), etc. If the factor of augmentation is three (a = 3) and therefore the intervals of the augmented series triple in size, the new series will be related to the initial one by the exponent of three. For example, compare the harmonic series (fundamental = C2) with the augmented series based on three octaves: C2, c, a1+6c, c3, c4-42c, a4+6c, f5+7c, c6, f#6+12c, c7-42c, f7-47c, a7+6c, c#8+20c, f8+7c, a8-36c, c9, etc. It follows that the lower the factor of augmentation, the larger the portion of common ("pivot") tones there is between the two series (practically speaking, between the equal segments[2] of the two series) and the easier it is to "modulate" from one series to another.

3.4 Transpositions

Each augmented version of the harmonic series can also be transposed. Figure 2 shows an augmented harmonic series (fundamental = C2) transposed by using notes of the chromatic scale in the tempered system as new fundamentals. For the purpose of modulating, the figure highlights "pivot tones" connecting the initial augmented series with its closely related transpositions ("related keys"). To be exact, it highlights transpositions based on the 2nd, 4th (2^2), 8th (2^3), 16th (2^4) and 32nd (2^5) note of the initial series (the order of notes is clearly related by powers of two). In Figure 2, I also outline the transposed series of which the fundamental is the third note of the initial augmented series. These transpositions were selected because comparatively to the relationship between the harmonic series and traditional harmony, they correspond to the tonic and dominant functions. Since each note of the series is a fundamental of a new series that is an exact transposition of the given series, and since any of these new series relates to the initial series by the order of n, 2n, 3n, 4n, 5n, etc., it follows that the smaller the n, the larger the portion of common notes there is between a transposed and a given series. Since the relationship between a given series and its transpositions is stronger when the fundamentals of the transpositions are the lower notes of the given series, the enclosed figure highlights "pivot tones" only in the transpositions based on the second and third note of the given series. (In the case of the transposition based on the third note of the initial series, one has to take into account the microtonal deviation of the new fundamental.) By applying the same process, one can easily find other links between

[2] Since all the series of the system are theoretically infinite, one can in practice apply only segments of the series and not, of course, the entire series.

the series. The transposition based on the second note of the given series presents 1/2 of the series, and the transposition based on the third note of the given series presents 1/3 of the series. On the other hand, the transpositions based on the 4th, 8th, 16th and 32nd "partial" of the initial series, are in Figure 2 not highlighted because of their quantitative value of common notes, but because of their significance in relation to equal temperament. While they present only 1/4, 1/8, 1/16, and 1/32 portion of the series, their fundamentals are always tempered pitches with no microtonal deviations. The number and structure of closely related transpositions vary from one augmented series to another, and one may choose to group augmented series based on their models of related transpositions. For example, there is an obvious parallel between transposition models found in the augmentations based on primary intervals of which the number of half-steps differs by multiples of twelve (for example: c1-c#2, c1-c#3, c1-c#4, etc.). It means that the transposition model of the augmented series illustrated in Figure 2 resembles a transposition model of the augmentations based on the primary intervals of 25, 37, 49, etc. half-steps. Likewise, transposition models of augmented series based on inverted primary intervals (± multiples of twelve half-steps) demonstrate similarities in structure. For example, the transposition model illustrated in Figure 2 is a mirror picture of the transposition model produced by the augmentations based on the primary intervals of 23, 35, 47, etc. half-steps.

3.5 Determining Frequencies

All the examples in the enclosed figures are expressed in cents. In order to express the notes of the augmented series in frequencies, one should use again the formula: $\log f_2/f_1 = c/3986$ (or $\log f_2 - \log f_1 = c/3986$). Likewise, the frequencies of the pitches in the augmented series are defined by the frequency of the fundamental multiplied by a serial number of the given note raised to a (where a is the augmentation of the augmentation). For example, if n again represents a serial number of a frequency and if intervals of the series, expressed in cents, are augmented by 5/4, it follows that the frequencies of the augmented series equal $f_1 \times n^{5/4}$. Similarly, when intervals are augmented by 2, the new frequencies equal $f_1 \times n^2$, or when intervals are augmented by 3, the new frequencies equal $f_1 \times n^3$, etc. (The last two examples also explain previously mentioned exponential relationships in integral augmentations.) In relation to the frequencies, one can conclude that the process of the augmentation applied in this system transforms the linear function determining the harmonic series into exponential functions determining the augmented series.

3.6 Analytical Model

In the lower half of Figure 1, I illustrate the way I analyze an augmented series before using it in a composition. Although my analysis is modeled after the presumed relationships between the harmonic series and the Western tonal system, I rarely apply such rigid concepts to my music. I prefer to explore unique characters of new music materials. The brief analysis in Figure 1 shows: the modes derived from the first sixteen notes of the series; the chords derived from the first six notes of the series; and quasi tonic-dominant progressions corresponding to the position of the implied tonic/dominant triads as found in the harmonic series. The fundamental of the series is treated as a quasi tonic. All mentioned modes and chords are always determined by both the octave repetitions and primary interval recurrences. As stated previously, the function of the primary interval recurrences can be in this system compared to the octave repetitions. As such, there are always at least two main interpretations of the pitches in the augmented series of the system. The first, more traditional, interpretation views octaves as doublings, while the second one treats primary intervals as such. Figure 1 concludes by expressing the intervals of the series in the numerical values that are more suitable for rhythmic/structural applications.

4 Conclusion and Future Directions

There are many ways of interpreting various parameters of my system. In relation to the registers, the note/segments of the series can remain in their initial form, or they can be transposed. Modes/chords (excluding primary interval recurrences, octave repetitions, both or neither) may or may not consider microtonal deviations. (Figure 3 illustrates the use of an augmented series in an orchestral medium where, for practical reasons, I avoid working with microtones. In order to compensate for the microtonal inaccuracies, I colored the higher notes of the series with glissandos in violins.) Chord progressions and modulations (to other transpositions, other augmentations or both) can be modeled after the Western tonal system (considering its presumed ties with the harmonic series) or they can exist independently. (Figure 3 illustrates quasi tonic-dominant progressions in lower strings and brass.) In addition, one can search for ties with various music systems created outside the Western music tradition and look for parallels between the proposed micro-tonal models and various non-tempered tuning systems practiced around the world. The mathematical structure of the augmentations can be used in relation to melody, harmony, rhythm, tempo and form. Most importantly, it can be used for pitch mapping and building new electronic sounds. One can continue the series above the 32nd "partial" and apply the process of the augmentation to the series of frequencies below the fundamental (augmented sub-inharmonic spectra). It is equally possible to create diminished forms of harmonic series and apply similar compositional strategies to a process of diminution.

Empirical studies could further evaluate our perception of "pseudo-octaves" and new consonant orders in the context of compositions where the spectrum is precisely linked to the tuning through the process of the augmentation described in this paper. It is my hypothesis that the interaction between the tunings of the given augmented series and their corresponding spectra shaping their timbres would tilt the consonant effect in favor of new primary intervals and create interval hierarchy unique to each individual augmentation. Mathews and Pierce's[3] (1980) limited testing of the stretched harmonic theory and, most importantly, groundbreaking research by Sethares[4], provide validity to my theory.

In *Augmentation Matrix,* the characteristics of the new series and the relationships between them are determined by the mathematical restructuring of the harmonic series. Acoustic results of different augmentations offer an endless number of unique sound structures, harmonies and compositional interactions allowing one to create new or "translate" old musical "languages." The system is a sort of numerological game or a matrix of infinite number of augmentations derived from the integer frequency ratios.

[3] "Our experiments do not decide finally among three views of harmony: that harmony depends on a fundamental bass or periodicity pitch (Rameau), that harmony depends on the spacing of partials (Helmholtz and Plomp) or that harmony is a matter of brainwashing." Matthews, Max V. & Pierce, John R. "Harmony and Harmonic Partials," *Journal of the Acoustical Society of America*, 68 (Nov. 1980):1252-1257.

[4] Sethares W.A. (2005): Tuning Timbre Spectrum Scale, Springer, London.

Fig. 1

Augmentation: x13/12

Primary Interval: 13 half-steps [m9]

Serial # (intervals)	Int. in cents x13/12	Half-steps + dif. in cents	Intervals	Example fund.=C2	Ser.#	Corrections (in cents)	Example with corrections
1	/	/	/	C2	1	0	C2
1-2	1300	13	m9	C2-Db1	2	0	C#1
2-3	760.5	8 (39.5c-)	m6-	Db1-A1*	3	39.5-	A1
3-4	539.5	5 (39.5c+)	P4+	A1-D*	4	0	D
4-5	418.1666667	4 (18.16c+)	M3+	D-F#*	5	18.16+	F#
5-6	342.3333333	3 (42.33c+)	m3+	F#-A*	6	60.5+/39.5-	A#
6-7	289.25	3 (10.75c-)	m3-	A-c*	7	49.75+	c/c#
7-8	250.25	3 (49.75c-)	m3-	c-eb*	8	0	d#
8-9	221	2 (21c+)	M2+	eb-f*	9	21+	f
9-10	197.1666667	2 (2.84c-)	M2-	f-g*	10	18.16+	g
10-11	178.75	2 (21.25c-)	M2-	g-a*	11	3.09-	a
11-12	163.5833333	2 (36.42c-)	M2-	a-b*	12	39.5-	b
12-13	149.5	1 (49.5c+)	m2+	b-c1*	13	10+	c1
13-14	139.75	1 (39.75c+)	m2+	c1-db1*	14	49.75+	c#1/d1
14-15	128.9166667	1 (28.9c+)	m2+	c#1-d#1*	15	78.7+/21.3-	d#1
15-16	121.3333333	1 (21.33c+)	m2+	d#-e1*	16	0	e1

Characteristics

1) Modes derived from the first 16 notes of the series; examples with "c" as fundamental.

A) Excluding primary interval [m9] recurrences (numbers indicate the position of the note in the series): c(1), c(13), c/c#(7), d#(15), f(9), f#(5), a-(3), a(11); 8 notes.

a) Number of recurrences: c(5x), c, c/c#(2x), d#, f, f#(2x), a, a-(3x).

b) Excluding notes deviating more than 25c*: c, c, d#, f, f#, a; 6 notes.

B) Excluding octave repetitions: c, c/c#, c#, c#/d, d, d#, e, f, f#, g, a-, a, a#, b; 14 notes.

a) Number of repetitions: c(2x), c/c#, c#, c#/d, d, d#(2x), e, f, f#, g, a-, a, a#, b.

b) Excluding notes deviating more than 25c*: c, c#, d, d#, e, f, f#, g, a, a#, b; 11 notes.

c) Excluding primary interval [m9] recurrences: c, c/c#, d#, f, f#, a-, a; 7 notes.

2) Chords derived from the first 6 notes of the series; examples with "c" as fundamental.

A) Excluding primary interval [m9] recurrences: c, f#, a; 3 notes.

B) Excluding octave repetitions: c, c#, d, f#, a-, a#; 6 notes.

3) Quasi tonic-dominant progression (numbers indicate the position of the note in the series and Roman numerals the position of the note in the triad); examples with C2 as fundamental.

A) T: 1-6, 8, 10, 12, 16; C2(I), C#1(I), A1-(III), D(I), *F#(I)*,A#(III), d#(I), *g(I)*,b(III), e1(I).

B) D: 3, 6, 9, 12, 15; A1-(I), A#(I), f(III), b(I), *d#(I)*.

4) Intervals of the series expressed in proportions more suitable for possible rhythmic/structural applications.

A) Intervals in cents/10: 130, 76, 54, 42, 34, 29, 25, 22, 20, 18, 16, 15, 14, 13, 12.

B) Intervals in cents/40: 30.5, 19, 13.5, 10.5, 8.5, 7.25, 6.25, 5.5, 5, 4.5, 4, 3.75, 3.5, 3.25, 3.

Fig. 2

Primary Interval: 13 half-steps [m9]

Ser.#	1... 0c	3... 40c-	5... 18c+	7... 50c+	9... 21c+	11... 3c-	13... 10c+	15... 21c-	Fund. = C2 (cents+/-)	Fund. C#2	Fund. D2	Fund. D#2	Fund. E2	Fund. F2	Fund. F#2	Fund. G2	Fund. G#2	Fund. A2 40-*	Fund. A#2	Fund. B2
1	C2								C2	C#2	D2	D#2	E2	F2	F#2	G2	G#2	A2	A#2	B2
2	C#1								C#1	D1	D#1	E1	F1	F#1	G1	G#1	A1	A#1	B1	C
3		A1							A1 (40-)	A#1	B1	C	C#	D	D#	E	F	F#	G	G#
4	D								D	D#	E	F	F#	G	G#	A	A#	B	c	c#
5			F#						F# (18+)	G	G#	A	A#	B	c	c#	d	d#	e	f
6		A#							A# (40-)	B	c	c#	d	d#	e	f	f#	g	g#	a
7				c					c (50+)	c#	d	d#	e	f	f#	g	g#	a	a#	b
8	d#								d#	e	f	f#	g	g#	a	a#	b	c1	c#1	d1
9					f				f (21+)	f#	g	g#	a	a#	b	c1	c#1	d1	d#1	e1
10			g						g (18+)	g#	a	a#	b	c1	c#1	d1	d#1	e1	f1	f#1
11						a			a (3-)	a#	b	c1	c#1	d1	d#1	e1	f1	f#1	g1	g#1
12		b							b (40-)	c1	c#1	d1	d#1	e1	f1	f#1	g1	g#1	a1	a#1
13							c1		c1 (10+)	c#1	d1	d#1	e1	f1	f#1	g1	g#1	a1	a#1	b1
14				c#1					c#1 (50+)	d1	d#1	e1	f1	f#1	g1	g#1	a1	a#1	b1	c2
15								d#1	d#1 (21-)	e1	f1	f#1	g1	g#1	a1	a#1	b1	c2	c#2	d2
16	e1								e1	f1	f#1	g1	g#1	a1	a#1	b1	c2	c#2	d2	d#2
17									f1 (14+)	f#1	g1	g#1	a1	a#1	b1	c2	c#2	d2	d#2	e2
18					f#1				f#1 (21+)	g1	g#1	a1	a#1	b1	c2	c#2	d2	d#2	e2	f2
19									g1 (22+)	g#1	a1	a#1	b1	c2	c#2	d2	d#2	e2	f2	f#2
20			g#1						g#1 (18+)	a1	a#1	b1	c2	c#2	d2	d#2	e2	f2	f#2	g2
21									a1 (10+)	a#1	b1	c2	c#2	d2	d#2	e2	f2	f#2	g2	g#2
22						a#1			a#1 (3-)	b1	c2	c#2	d2	d#2	e2	f2	f#2	g2	g#2	a2
23									b1 (20-)	c2	c#2	d2	d#2	e2	f2	f#2	g2	g#2	a2	a#2
24		c2							c2 (40-)	c#2	d2	d#2	e2	f2	f#2	g2	g#2	a2	a#2	b2
25									c2 (36+)	c#2	d2	d#2	e2	f2	f#2	g2	g#2	a2	a#2	b2
26							c#2		c#2 (10+)	d2	d#2	e2	f2	f#2	g2	g#2	a2	a#2	b2	c3
27									d2 (19-)	d#2	e2	f2	f#2	g2	g#2	a2	a#2	b2	c3	c#3
28				d2					d2 (50+)	d#2	e2	f2	f#2	g2	g#2	a2	a#2	b2	c3	c#3
29									d#2 (15+)	e2	f2	f#2	g2	g#2	a2	a#2	b2	c3	c#3	d3
30								e2	e2 (21-)	f2	f#2	g2	g#2	a2	a#2	b2	c3	c#3	d3	d#3
31									e2 (40+)	f2	f#2	g2	g#2	a2	a#2	b2	c3	c#3	d3	d#3
32	f2								f2	f#2	g2	g#2	a2	a#2	b2	c3	c#3	d3	d#3	e3

Selected relationships between transpositions

x x x x *Represents 1/2 of the given series.*

x x x *Represents 1/4 of the given series.*

x x *Represents 1/8 of the given series.*

x *Represents 1/16 and 1/32 of the given series.*

x x *Represents 1/3 of the given series (with additional micro-tonal deviations determined by the fundamental).*

Fig. 3

excerpt from "Ashen Time"
(score in C)

BIBLIOGRAPHY

P. Cariani, “A temporal model for pitch multiplicity and tonal consonance,” ICMPC (2004).

E. C. Carterette and R. A. Kendall, “On the tuning and stretched octave of Javanese gamelans,” Leonardo Music Journal, Vol. 4, 59-68 (1994).

J. Douthett, R. Entringer, and A. Mullhaupt, “Musical scale construction: the con-tinued fraction compromise,” Utilitas Mathematica 42, 97-113 (1992).

J. M. Geary, “Consonance and dissonance of pairs of inharmonic sounds,” J. Acoust.Soc. Am. 67, No. 5, 1785-1789 May (1980).

R. J. Krantz and J. Douthett, “A measure of reasonableness of equal-tempered mu-sical scales,” J. Acoust. Soc. Am. 95, No. 6, 3642-3650, June (1994).

M. V. Mathews and J. R. Pierce, “Harmony and inharmonic partials,” J. Acoust.Soc. Am. 68, 1252-1257 (1980).

T. D. Rossing, The Science of Sound, Addison Wesley Pub., Reading, MA (1990).

J. Sankey and W. A. Sethares, “A consonance-based approach to the harpsichord tuning of Domenico Scarlatti,” J. Acoust. Soc. Am. 101, No. 4, 2332-2337, April (1997).

W. A. Sethares, “Local consonance and the relationship between timbre and scale,”J. Acoust. Soc. Am. 94, No. 3, 1218-1228, Sept. (1993).

W. A. Sethares, “Relating tuning and timbre,” Experimental Musical Instruments IX, No. 2 (1993).

W. A. Sethares, “Adaptive tunings for musical scales,” J. Acoust. Soc. Am. 96, No.1, 10-19, July (1994).

W. A. Sethares, “Tunings for inharmonic sounds,” Synaesthetica ACAT, Canberra, Australia, Aug. (1994).

W. A. Sethares, Tuning, Timbre, Spectrum, Scale, Second Edition, Springer Verlag, London (2004).

W. A. Sethares, “Specifying spectra for musical scales,” J. Acoust. Soc. Am. 102, No. 4, Oct. (1997).

W. A. Sethares, “Consonance-based spectral mappings,” Computer Music Journal22, No. 1, 56-72, Spring (1998).

W. A. Sethares, “Real-time adaptive tunings using MAX,” Journal of New Music Research 31, No. 4, Dec. (2002).

W. A. Sethares and B. McLaren, “Memory and context in the calculation of sensory dissonance,” Proc. of the Research Society for the Foundations of Music, Ghent, Belgium, Oct. (1998).

W. A. Sethares and T. Staley, “Sounds of crystals,” Experimental Musical Instru-ments VIII, No. 2, Sept. (1992).

A. Storr, Music and the Mind, Ballantine Books, New York (1992).

W. J. Strong and G. R. Plitnik, Music, Speech, Audio, Soundprint, Provo, UT (1992).

J. Sundberg, The Science of Musical Sounds, Academic Press, New York (1991).

M. Tenzer, Balinese Music, Periplus Editions, Inc. (1991).

N. Vicentino, “L’antica musica ridotta alla modernaprattica,” Antonio Barre, Rome (1555), English translation: “Ancient music adapted to modern practice,” eds. M. R. Mani-ates and C. V. Palisca, Yale University Press, New Haven, CT (1996).

H. Waage, “Liberation for the scale-bound consonance,” 1/1 7, No. 3, March (1992).